The Role of the Abandonment Option in Strategic Capital Allocation: A Review of Selected Literature*

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To cite this article: Ribeiro, J. A. & J. Costa-Filho. 2022. The Role of the Abandonment Option in Strategic Capital Allocation: A Review of Selected Literature. European Review of Business Economics I(2): 29-58; DOI: https://doi.org/10.26619/ERBE-2021.I.2.3

ABSTRACT

We review the most relevant contributions to the abandonment option since the late 1960s. We begin by approaching the contributions to the literature before the emergence of the real options approach to capital investment decisions, and thereafter, under a consistent real options approach, highlighting the interactions between the option to abandon and other types of options. We then identify the methodologies adopted, and the business sectors/ types of investment projects where the abandonment option is more frequently studied. We also debate the strategic role of the abandonment solution in corporate divestitures and under a game-theoretical approach. Finally, we present some concluding remarks and identify how certain gaps found in the literature may constitute opportunities for future research.

Keywords: Real Options; Abandonment Option; Investment Projects; Resource Allocation; Divestitures; Strategic Decisions. Real Options Games.

JEL Codes: D81; G31.

THE APPLICATION OF THE OPTION PRICING THEORY to the valuation of investment projects was triggered by Myers (1997), who stated that investment opportunities may be viewed as 'Real Options' since management has the right but not the obligation to acquire real assets on possible favorable terms. From then onwards, a substantial number of researchers have contributed to what is one of the most crucial landmarks in modern finance theory – the Real Options Approach (ROA) to capital investment decisions and resource allocation.

* The authors would like to thank Mário Coutinho dos Santos and two anonymous referees for their useful comments and suggestions. All errors and omissions remain ours. The authors did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors, for this research.

The ever-increasing advancement of technology which has led to a more rapid obsolescence and shorter life cycles and the fact that competition has become more intense have resulted in an increased focus on the abandonment decision in the analysis of investment projects. Airlines frequently close routes where the demand is insufficient to make the operation profitable. Real estate projects are abandoned during construction and R&D processes are shut down in their early stages. Natural resources companies decide to close mines (for example) when the price of commodities falls to a level where the expected cash flows become smaller than the abandonment value and, sometimes, even negative. Microsoft permanently closed some of its stores in 2020 and, in the same year, Walmart temporarily closed 60 stores due to the lack of demand caused by the COVID 19 pandemic. Likewise, Boeing announced a temporary suspension of production operations at its Puget Sound area facilities considering the state of emergency declared in the Washington state because of the turbulence caused by the COVID 19 pandemic, also in 2020. These are recent real-life cases illustrating the relevance of exercising the option to (temporarily or permanently) abandon activities.

The consideration of the possibility of abandoning a project and the recognition that such an option generates value may be found in research published years before Myers's (1977) work set the foundations for the emergence of a consistent ROA to capital allocation decisions.

The article by Robicheck & Van Horne (1967) was the first to stress the importance of considering the value of abandoning a project. The authors argue that a project should be abandoned at the point in time when its salvage value – the market value of an asset in its next most productive use – is greater than the Net Present Value (NPV) of all expected cash flows discounted using a discount rate that expresses the cost of capital. It was the first piece of research to present a decision rule to determine whether (and when) a project should be abandoned at various stages between the asset's acquisition and the end of its useful life. However, a subsequent comment from Dyl & Long (1969) claimed that the proposed decision-making criteria did not consider all possible cases of abandonment over the life of the asset. The latter authors argued that the abandonment decision rule should allow managers to choose between three options[1]: *(i)* the option to hold on to the project; *(ii)* the option to abandon it now, and *(iii)* the option to abandon the project at some point in the future. In a reply to this theory, Robicheck & Van Horne (1969) agreed that their model should be extended to accommodate the possibility of delaying the abandonment to a future date.

Later, Herbst (1976) further extended the model by relaxing one of the assumptions, hence considering that management may be faced with capital restrictions, which implies that it may be optimal to abandon a project when other projects in hand exhibit a greater expected profitability. Moreover, Herbst (1976) was the first to explicitly assess the value of the option to abandon when computing

[1] Interestingly, the authors used the word 'option' in the text, years before Myers (1977) coined the expression 'real options'.

the project's expected NPV. Prior research was mostly concerned in deriving a decision rule to determine the optimal abandonment after the project had been selected and was already active. On the other hand, Herbst's approach focuses on the impact of what he refers to as the 'intermediate salvage value', since it affects the initial decision concerning which (if any) of an array of mutually exclusive projects should be selected in the first place. This author proposes a method of analysis that may be repeated in subsequent periods to incorporate revised estimates of project characteristics, as a result of the arrival of new relevant information. Bonini (1977) applied a Dynamic Programming approach to construct the entire abandonment strategy, by determining a policy that specifies how far cash flows can deviate from the initial estimate before early abandonment should be adopted.[2]

The above-mentioned research papers were crucial in establishing and enforcing the importance of considering the existence of flexibility when appraising and managing active capital investment projects under uncertain conditions. Until then, management's attitude during an active project was passive – once a project was selected and implemented, management would merely watch the project generating cash flows for its useful life, and the NPV would be computed by deducting the sum of the present value of all cash flows to the cost of all investment expenses. At the project's appraisal stage, the expected NPV would be calculated assuming that the project would follow its predefined course, hence not considering the value of the managerial flexibility to alter the project's route whenever optimal conditions were met. Thus, available options to alter the project's initial conditions, such as the option to delay the project implementation, the option to switch inputs or outputs, and the option to abandon or downscale a project were not included. Within such a context, the option to abandon or shut down an entire project is one of the most important real options that management should assess and incorporate in their decision-making process, notably in times when uncertainty levels impacting the project's expected cash flows are perceived as being extremely high. This paper also aims to emphazize the strategic importance of including the value of the option to abandon when estimating the project value and to describe the main methods that enable managers to value such an option before projects are selected and when projects are already active. We also aim to identify the business sector/type of investments in which the option to abandon appears to be of greater importance, and to contribute to the existent body of real options literature by identifying some gaps that may constitute opportunities for future research.

The remainder of this paper is structured as follows. In Section 1 we cover the most relevant theoretical contributions to the abandonment option under a real options framework. Section 2 is dedicated to describing the relevant methodologies applied to assess real options and the option to abandon. In Section 3, we review the most relevant contributions to the literature with an emphasis on the natural resources industry and R&D, innovation projects, and technology

[2] We debate the dynamic programming approach in Section 2.

adoption and, finally, on infrastructure projects. In Section 4, we approach the strategic role of real options and the strategic interactions of the real options theory with the strategic management literature, firstly considering the importance of corporate divestitures and, secondly, under a game-theoretical approach. Finally, in Section 5, we present concluding remarks and make some suggestions in terms of future research.

2. The Option to Abandon Under the Real Options Approach

The similarity between real investments and financial investments' decision--making has been recognized for at least four decades, since researchers such as Tourinho (1979), Myers & Madj (1983), McDonald & Siegel (1985) and McDonald & Siegel (1986) extended the financial option theory of Black & Scholes (1973) and Merton (1973) to include irreversible real investments, when flexibility and uncertainty features are of critical importance. We adopt the argument by Kumar (2016) and Amram & Kulatilaka (2000), and state that the real options theory is essentially an extension of the financial options theory because a real option is nothing more than the flexibility a manager has for making decisions about real assets (Sick, 1995). Thus, it becomes clear that it is possible to establish an analogy between capital investment decisions and those concerning financial options. The role of uncertainty, the presence of (at least, partial) irreversibility and the recognition of flexibility in the decision-making process are very similar in both cases. For a decade immediately after the appearance of the option pricing model, several studies were presented with the purpose of applying the option pricing technique to solve the valuation problems of various financial instruments such as convertible bonds, warrants, stocks, and insurance contracts. In the late 1970s and in the 1980s, the application of option pricing started to expand beyond the limit of financial instruments to include some economic problems that have the option-like structure. Tourinho's work (1979) is considered to be pioneering in establishing this analogy, in the context of the valuation of the extraction of natural reserves. In his work, the author clearly states that "to calculate the value of the reserve under uncertainty I view it as an option to extract the resource in the future. Seen in this way, the reserve is analogous to a (call) option on a stock" (p. 11). Moreover, he argued that" the owner of the reserve possesses an option in which the underlying asset is the resource. More precisely the reserve is an American option, which is like a European option, except that it allows exercise prior to expiration date" (p. 12). The consequences of this analogy were of great importance to the establishment of the ROA to capital allocation. Furthermore, Tourinho (1979) was the first to use a continuous-time stochastic approach to model the stochastic behavior of the 'underlying asset' – the present value of cash flows to be generated by exploring the reserve, a diffusion process known as geometric Brownian motion (gBm).

Table 1 shows the analogy between a financial put option and the option to abandon an investment project.

Table 1
Analogy between a Financial Put Option and the Option to Abandon

Financial Put Option	Option To Abandon
Price of the Underlying Asset	Present Value of the Project's Cash Flows
Exercise Price	Project's Salvage Value
Time to Maturity	Remaining Life of the Project
Volatility of the Underlying Asset	Volatility of the Project's Future Cash Flows
Risk-Free Interest Rate	Discount Rate
Dividend Yield	"Convenience Yield"

Other early contributions to incorporate the value of the option to abandon in capital budgeting decisions are the ones by Kensinger (1980), Myers & Madj (1983), De-et-al. (1983), Cox & Martin (1983), Brennan & Schwartz (1985) and McDonald & Siegel (1985). Kensinger (1980) proposed a model with practical limitations since the option to abandon possesses a finite life, like a European put option on a stock. His approach decomposes the life of the project into two periods: the period prior to the expiration of the abandonment option while the second period is the remaining life of the project after the expiration of the abandonment option. The well- known Black & Scholes (1973) formula is applied to treat a somewhat restrictive problem, since the abandonment decision must be made on a specific predefined date.

However, the research by Myers & Madj (1983) (revised in 1985 but only published in 1990[3]) is particularly important in the context of this section because their model assumes that the abandonment option may be exercised at any moment in time. The authors have determined the abandonment value by applying the option pricing theory and, to the best of our knowledge, it is the first piece of research to assess the value of the option to abandon a project under rational option pricing theory or, which is analogous, under the ROA. The authors state that the option to abandon is formally equivalent to an American put option on a dividend paying stock, in which the exercise price of the put option is the salvage value of the project's assets, and the cash flows to be generated by the project are equivalent to the dividend payments on the stock (as established by the classical model of stock valuation).

Myers & Madj (1983) were the first authors to introduce the bidimensionality in the optimal decision rule, since two stochastic variables are considered. Since then, this dimensionality issue has proven to be a complex one. There are some situations in which one can reduce the bidimensionality of the model to

[3] Please see Myers & Madj (1990).

just one variable by substituting the two variables with a ratio between them. This is obviously a procedure that implies that a relationship may be established between the two variables, in this case, the salvage value and the project value. It is easy to agree that these two variables are correlated in some manner, however stating the existence of a deterministic relationship is more complex. Myers & Madj (1983) draw attention to this argument and point out that a complete specification of the stochastic properties of the salvage value, including its relation to project value, is not an easy task. They present a solution by resorting to numerical methods and, more specifically, to an explicit finite difference approximation. Their analysis demonstrates that, other things being equal, the value of the abandonment option increases with the salvage value (the 'exercise price'), project volatility, and with project life (time.to-maturity), and decreases with project value, in line with the predictions of the put option pricing theory.

De-et-al. (1983) contributed to the existent literature by recognizing the stochastic nature of the salvage value and the life of the project, an aspect disregarded by Myers & Madj (1983), since they implicitly assumed the abandonment option to be perpetually available.

Previous authors have emphasized the need to consider the probabilistic nature of the project life (see, for example, Wagle, 1967; Van Horne, 1972; Hertz, 1979), even though none of them have treated such a feature in their models. De-et-al. (1983), on the other hand, suggested a model in which both the salvage value and project life are stochastic variables, and presented a solution obtained using Dynamic Programming and applying the multi-period Capital Asset Pricing Model (CAPM) solution proposed by Myers & Turnbull (1979). Cox & Martin (1983) refer to the 'capital deepening problem', an expression used to describe the decision that must be made in any investment related to the optimal term to hold the asset, *i.e.*, the mirror image of the abandonment problem. They then proposed a solution in which the net cash flows, the salvage value and the rate of discount expressing the opportunity cost of capital are stochastic. These authors were able to incorporate risk-aversion in the model by considering the stochasticity of the discount rate, which decreases over time.

The work by Brennan & Schwartz (1985) is one of the most prominent pieces of research in the early days of ROA. The authors derive a model in which one commodity (copper) may be extracted from a mine and consider the commodity's price to be stochastic and modeled as a gBm. Firstly, they consider the mine capacity to be a random variable that can assume two states: *i)* the mine has inventory to extract; *ii)* the mine is exhausted. They later present an extension to the model and consider the mine to have an infinite capacity, hence, being inexhaustible, and arrive at a solution that allows managers to optimally decide to close the mine if the output price falls below a threshold level and reopen it when the output price reaches another threshold price level and assume that closing and reopening the mine bears a cost, which is equal in both cases. The option to permanently close the mine is also addressed and the authors determined this option's value for different output prices.

Brennan & Schwartz (1985) addressed the entry-exit decision for the first time: firms may suspend (or mothball) operations for some time and eventually reopen them later. As we will see, later other authors have embraced this possibility of temporary abandonment and subsequent reentry (and eventually closing again, and so forth), such as Dixit (1989), Dixit (1992) and Dixit & Pindyck (1994).

The paper by McDonald & Siegel (1985) assumed considerable importance in the early stages of the ROA. These authors addressed the abandonment option by considering that a firm has the option to shut down the production if variable costs exceed revenues. The output price and the output costs are each governed by a gBm, and the project value is a function of these two state variables, hence assuming a bidimensional nature, which is reduced to just one dimension by merely substituting the two variables by one – a ratio between the two – and, therefore, obtaining an analytical solution. The model also contemplates the risk-averse factor, presenting results that are different from the ones obtained under the standard 'risk-neutrality' assumption.

Table 2 summarizes the features of the most relevant contributions to the abandonment option discussed in the current section.

Table 2

Early Contributions to the Abandonment Option under the ROA

Author(s) (Year)	State Variables(s)	Model Features
Kensinger (1980)	1. Project value	Continuous time: variable follows a gBm
Myers & Madj (1980)	1.Project value; 2. Expected dividends; 3. Salvage value	Continuous time: variables follow gBm's
De et al (1983)	1.Project value; 2. Salvage value	Discrete time and Dynamic Programming
Cox & Martin (1983)	1.Project value; 2. Salvage value; 3. Discount Rate	Continuous time: variables follow gBm's
Brennan & Schwartz (1985)	1.Output price; 2. Mine inventory	Continuous time: variable 1 follows gBm; variable 2 is binary
McDonald & Siegel (1985)	1.Output price; 2. Cost per unit of output	Continuous time: variables follow gBm's

In 1994, Avinash Dixit and Robert Pindyck published a textbook entitled 'Investment under Uncertainty', which has become a fundamental reference in the real options literature. Dixit & Pindyck (1994) addressed the abandonment option in chapter 7 of the book, firstly by considering a combined entry and exit strategy model in the presence of exit costs and reentry costs – in previous chapters, the entry-and-exit strategy was addressed, albeit assuming that there were no costs associated with any of the decisions. These researchers also put forward another extreme scenario: if the operations are ever suspended, the firm must incur in the whole initial investment cost again to restart them; however, the authors consider this extreme scenario unrealistic and, in most instances, the most frequent cases lie somewhere between the extreme alternatives: firms incur in reentry costs, but usually they are not as large as the initial investment outlay. Finally, instead of suspending operations, the firm must contemplate the possibility of definitive abandonment. Since restarting is costly, there is an option value of keeping the operations alive, and abandonment will only be optimal when the level of operating losses is large enough to overcome the restarting costs.

In this line of reasoning, Dixit & Pindyck (1994) address the following theoretical problems: *(i)* to entry and exit operations, deriving solutions for both the idle firm and the active firm, and using a continuous-time state variable and a stochastic variable in discrete time, which assumes the value of 1 if the firm is active and the value of 0 if the firm is idle. The model allows managers to determine the low level price threshold that prompts the active firm to suspend operations and the high level price threshold that triggers the reentry decision of the idle firm, in the presence of exit and reentry costs (see also Dixit, 1989); *(ii)* lay-up and reactivation, since firms may temporarily suspend operations (for example, ships are' laid-up' when freight rates are not sufficient to cover operational costs, and plants are 'mothballed' for similar reasons) and reactivate them later, when optimal conditions are met[4]. *(iii)* scrapping, when firms exercise the option to abandon operations permanently.

The differences between the three above-mentioned strategies are somewhat subtle. As we mentioned, in the first case the authors derive two threshold levels, the level where it is optimal for an active firm to exit the market for a given amount of abandonment costs, and the price level that prompts an idle firm to reenter the market, for the same amount of abandonment costs.[5] Also, the price level that triggers the firm to enter a mothball state considering the corresponding costs of mothballing is derived. To reach this threshold, the authors consider

[4] The fist paper dedicated to the lay-up/ reactivate problem is Mossin (1968), before the Real Options Approach was established.

[5] When the output price is located somewhere between these two triggers, further analysis is needed since the current level of the output price is not sufficient: we need to know the previously state of the firm. If the output price is at its current level after recently descending from a high level that prompted entry, then we should expect to encounter an active firm. On the contrary, if this output price intermediate level was recently preceded by a low level that triggered the option to exit, then we will expect the firm to stay idle. This means that the economy is 'path dependent'. There is a significant body of research dedicated to the path dependency of economic problems. In our context, this path dependence can lead to the following situation: a firm is contemplating investment and the current output price (or profit) is in the intermediate range between the low-level threshold and the high-level threshold. Therefore, the firm's optimal decision is to wait and see. Then, if the output price rises over the high threshold, the firm decides to invest. Finally, if the price falls to the previous lower level, but still above the low-price trigger, the firm will not exit the market. Hence, the underlying cause (current price) has been restored to its old level, but its effect (investment) has not. This phenomenon is known as 'hysteresis', and by analogy the failure of investment decisions to be reversed when the underlying causes are fully reversed can be called 'economic hysteresis' (Dixit & Pindyck, 1994, p. 27).

a third state for the above-mentioned selected discrete time variable – the mothballed state. Finally, a fourth trigger is derived, which is the output price level that triggers the firm to scrap, *i.e.*, to permanently abandon operations. The four thresholds are derived applying numerical methods, as they do not have analytical solutions [6].

This approach also entails another aspect that we would like to highlight. Whenever a firm exercises the option to exit, this decision grants it the option to reenter again; whenever a firm reenters the market, the firm receives the option to exit again, to mothball operations or permanently shut them down. Likewise, whenever a firm enters a mothball state, it may decide to reopen operations, it also then has the option to mothball operations again and to permanently abandon activities. This is a compound options problem, because when a firm exercises one option, it has at least another option that may be exercised in the future if optimal conditions are met. Therefore, a 'collection' of options is available.[7]

If, in theoretical terms, to separate the option to abandon from the others seems somewhat restrictive, in practice research mostly treats options separately or in pairs due to the mathematical complexity involved by addressing more than two options. However, the so-called sequential investment problem is also subject to considerable attention by real options researchers. Sequential investment has a cascading option structure: there is an initial investment outlay, and the first stage of the investment is completed. Then, as new information arrives, firms decide to abandon the project or to keep on investing. As always, the optimal decision is made considering the salvage value of the project at that time and the present value of the expected benefits it may generate in the future[8]. Sequential investment focuses on post-investment abandonment, and the option to abandon after investment expenses have ended is widely studied in the real options literature. However, an abandonment option can exist before the investment is undertaken, which we define as pre-investment abandonment (the terminology applied by Adkins & Paxson, 2017). These authors also state that the abandonment option for realizing the after-use resale value of project assets in scrap-metal and second-hand markets due to deteriorating conditions is well recognized, but pre-investment optionality is rarely examined despite its relevance in cases such as the sale of technological and R&D patents instead of their exploitation, or the sale of vacant plots instead of real estate developments.

When reviewing the selected literature in section 3, we will specifically mention the additional options (if any) addressed in each piece of research.

[6] Please note that exiting and reenter costs are assumed to be of the same magnitude, but the mothball costs and scrapping costs are of a different nature.

[7] To discuss the mathematical treatment of compound options is beyond the scope of this work. However, the reader may wish to refer to the important contributions by Geske (1979) and Carr (1988).

[8] We will discuss the sequential investment problem in more detail in section 3.

3. Methodologies to Valuing Real Options

The two methodologies applied in real options valuation over the last four decades can be classified based on two types of stochastic processes and methods: analytical continuous-time processes, which use Analytical Methods (AM) and Numerical Methods (NM). Both AM and NM can compute the option's value by using either Contingent Claim Analysis (CCA) or Dynamic Programming (DP). CCA assumes that markets are complete and, therefore, it is possible to replicate the stochastic behavior of the project's cash flows or the output price through the construction of a portfolio of securities trading constantly in the financial markets (Trigeorgis & Mason, 1987; Dixit & Pindyck, 1994; Alexander & Chan, 2021).This is the so-called replicating portfolio assumption', by which the project value must equal the value of a portfolio of traded assets with the same cash flows, otherwise one could profit from taking an arbitrage opportunity. To be more specific, since the risks of the replicating market portfolio and the project are assumed to be the same, the value of the two assets must be the same as well. Therefore, CCA is applicable only when financial assets and options result in a complete set of contingent claims on income across all possible market states. Therefore, CCA assumes that there is a traded asset whose fluctuations are perfectly correlated with the stochastic process of the state-variable, as stated by Dixit & Pindyck (1994) (p. 136).

The assumption of complete markets is of utmost importance because traded assets need to be combined to yield any combination of payoffs across any possible sets of states of nature. However, in practice, markets are not complete (see, for example, Branger at al., 2018, Delaney, 2020) and such an assumption becomes restrictive, thus limiting the widespread applicability of CCA in real-life project valuation. Many real projects are exposed to incomplete markets for project-specific risk dynamics. In these cases, DP is applied to find the optimal value for an investment decision problem under uncertainty by maximizing the project's NPV while also accounting for the presence of flexibility embedded in capital investments. The dynamic optimization problem is divided into simple subproblems, as prescribed by Bellman's principle of optimality. For instance, in a finite-time setting, the mechanics underlying DP can be looked at as a backward induction process that is conducted from the terminal decision point until the problem's starting point.

To track the evolution of a project's value using DP, the risk variables can be modeled as continuous-time processes or discrete-time processes. In the latter, the state variables are defined to completely characterize the asset's market returns and it is assumed that they follow a Markov process, which is in fact a discrete-time analogous to the continuous-time gBm process. Then, at each step, the decision related to the asset's operation (which is a control variable) influences the alternative paths taken by the state variable, meaning that the project value and the value of the option are functions of both variables, *i.e.,* the control variable and the state variable. The option value is then maximized using a Bellman equation or, within a continuous-time framework, a partial differential equation called Hamilton-Jacobi-Bellman (HJB) equation (see Candler, 2001 for the numerical solution of a HJB equation using the finite-differences method). Using DP implies having to work with an exogenous and arbitrary discount rate, which in practice should express the opportunity cost of capital (Dixit & Pindyck, 1994). Due to its complexity, the application of DP techniques has remained limited (see, for example, Borinson, 2005; Triantis, 2005) and for this reason researchers seem to prefer using CCA, hence avoiding the problem of defining arbitrary discount rates.

The real options literature includes a substantial amount of research where probabilistic methods are used.[9] The binomial model suggested by Cox et al. (1979) and applied by many authors (*e.g.*, Xiaoran, 2020; Rambaud & Perez, 2016; Ulrich, 2013) and other more sophisticated lattices (e.*g.,* Song et al., 2017, Zhou & Cao, 2020), in addition to the Monte Carlo simulation (e.*g.,* Amédèe-Manesme et al., 2013; Kryzia et al, 2020; Maier, 2021), have been largely applied to model capital investment problems in different business sectors and industries. Nevertheless, continuous-time models, where state variables follow some type of diffusion process, have also been widely applied since the early days of the ROA, and the following processes are the ones most frequently used to model the state variable(s): (*i*) geometric Brownian motion (gBm); (*ii*) Poisson or jump process; (*iii*) gBm with jumps; (*iv*) Mean reversion.

The decision about which methodology to apply is not always an easy one. The mathematical complexity associated with derivatives and real options theory is caused by the need for a probabilistic solution to the optimal investment decision throughout the life of the option. The solution to this dynamic optimization problem, as described by Dixit & Pindyck (1994), is to model the uncertainty of the underlying asset as a stochastic process where the optimal decision value of investment is obtained by solving a differential equation with the appropriate boundary conditions. In many cases, however, this differential equation has no analytical solution, since one may define two or more stochastic variables in the model and the boundary conditions of the corresponding partial differential equation do not allow for homogeneity of degree one, or because considering just one stochastic variable does not reflect the actual complexity of the problem in hand.

Adkins & Paxson's (2011) quasi-analytical solution needs to be discussed since it was the first attempt to derive a two-factor uncertainty model that could be applied when the boundary conditions of the differential system infringe homogeneity of degree one. The authors constructed a set of simultaneous equations where the number of variables is greater than the number of equations. Since the authors' purpose was to reach a free boundary solution and not a single point threshold, this model's indeterminacy was irrelevant. However, the promise of finally obtaining an analytical solution to a two-factor uncertainty problem

[9] Mun (2006) offers a complete view of the different methodologies applied and presents many practical applications and real-life cases. Copeland & Antikarov (2003) also provide a good range of numerical methods applied to valuing both financial options and real options.

modeled in continuous time did not last long. Lange et al. (2020) stressed the inadequacy of the model and demonstrated that Adkins & Paxson (2011) solution is incorrect because it leads to suboptimal decisions. Therefore, this type of continuous-time approach based on multiple sources of uncertainty continues to have no analytical solution. Researchers need to keep on resorting to numerical methods or a discrete approximation to the underlying stochastic process may be used to obtain a solution that is computationally efficient. The Monte Carlo simulation is especially effective in these cases due to the increasing computational capacity shown by the latest simulation software versions. Harikae et al (2021) state that Monte-Carlo simulation is an alternative to lattice-based approaches when options are of European-style. For more complex American-style options with potential early exercise dates, Longstaff & Schwartz (2001) proposed a hybrid approach combining the Monte Carlo simulation and the functional approximations of probability density of continuing values using the Least-Square Method (LSM), which was also applied by Sabour & Poulin (2006), Hahn & Dyer (2011), Abadie & Chamorro (2017) and Harikae et al (2021).

In the next section, we review selected literature for the most relevant industries/types of investment projects where ROA is applied. In the Appendix, we provide the reader with information concerning the methodology used in each piece of work, emphasize the nature and the diffusion process of the corresponding state variable(s) and state whether the option was treated individually or in conjunction with other options[10].

4. Review of Selected Literature

In the present section, we review some of the most relevant contributions to the abandonment option within the sectors/types of investment projects where flexibility and uncertainty are of utmost importance and investment irreversibility is present. More specifically, we will address the natural resources industry, R&D and innovation projects and technology adoption, and infrastructure projects.

4.1. The Natural Resources Industry

The natural resources industry is characterized by high levels of volatility in the output price, *i.e.*, the price of commodities, most of them traded in the futures market. This fact makes the sector particularly eligible for the application of the ROA since flexibility has more value when output prices are more volatile. Furthermore, CCA is particularly attractive since the 'self-replication portfolio' assumption becomes more realistic.

Natural resource investments typically contain many interacting flexible components, and their performance is generally affected by multiple sources of

[10] More specifically, the Appendix contains the following information: (i) the output treated, when applicable; (ii) the state-variables(s); (iii) the diffusion process(es); (iv) formalism adopted; (v) methodology applied; (vi) additional options treated, if any; (vi) industry/type of investment project.

uncertainty. As we have mentioned, Brennan & Schwartz's (1985) original copper mine example considered a set of options: to mothball (temporary suspension) the mine, to reenter again and then to irreversibly abandon the project. The work of Brennan & Schwartz (1985) was not only one of the first to consider multiple options, but it also accounted for copper price uncertainty directly, rather than through a (single) risk-adjusted discount rate. Building upon their simple one-factor, constant-convenience model, where the price dynamics are described by a gBm, subsequent works aimed at accounting for the mean-reverting tendency of many commodities' spot prices. In the joint stochastic process of the two-factor model of Gibson and Schwartz (1990), the convenience yield evolves randomly by following an Ornstein-Uhlenbeck's mean- reverting process, and the three-factor model of Schwartz (1997) extends this two-factor uncertainty model by assuming that the risk-free interest rate follows a simple mean-reverting process.

Other relevant contributions to temporary or permanent abandonment in the natural reserves industry are the ones by Olsen & Stensland (1988), Clarke & Reed (1990), Cortazar et al. (2001), Lumley & Zervos (2001). Olsen & Stensland (1988) developed a model that can be applied to any type of commodity, and they assume that both price and quantities of the reserve are stochastic variables governed by gBm. They proposed a solution for the optimal decision to permanently abandon operations by deriving the optimal stopping rule where, at a certain moment, it becomes optimal to close the activities since the expected value of profits has reached its maximum. Clarke & Reed (1990) also defined the price and quantities as state variables but transform them into just one variable – the revenue, and through analytical methods, they reach a closed-form solution for the optimal abandonment problem, in line with the one proposed by Myers & Madj (1983). Cortazar et al. (2001) suggest a model that incorporates geological and technological uncertainty into one risk factor assuming it follows a simple Brownian motion. The output price is the other state variable, governed by a gBm, and the authors resort to numerical methods to obtain price triggers for the option to invest sequentially and, also, to permanently close operations. Lumley & Zervos (2001) derive a model that addresses the entry-exit decision. They assume that the any commodity price stochastic behavior is governed by a gBm and define another state variable, of a binary nature: the firm may be active or closed and reach an analytical solution through Dynamic Programming.

The most recent contributions are the ones by Guedes & Santos (2016), Abadie & Chamorro (2017), Zhou & Cao (2020) and Maier (2001). Guedes & Santos (2016) use a set of sophisticated lattices to appraise the value of several options, including the abandonment option. Based on a real-life case, the study illustrates the application of ROA to an oil field development project. Results show that real options' value turns a project destined to rejection into a project with positive economic value. The reversal is largely due to the option to abandon, which allows for project exit when exploration and appraisal activities yield poor outcomes, or the price of oil becomes excessively low. Abadie & Chamorro (2017) also approach an investment problem in the oil sector under uncertainty concerning the crude oil price, cost per unit and the reserve quantity. They derive optimal solutions to invest, to mothball and to permanently abandon operations. They first consider stochasticity in the output price and cost per unit to reach a free boundary that separates the waiting region from the investing region, a bidimensional solution that follows the solution suggested by Dixit and Pindyck (1994) since the boundary conditions do not infringe homogeneity of degree one. Later in the paper they also derive solutions to the mothball decision and the abandonment option, resorting to the Monte Carlo simulation. Zhou & Cao (2020) apply a trinomial lattice to assess the option to abandon an overseas oilfield project, in the presence of multiple sources of uncertainty – oil price, exchange rate, political environment and taxation policy. They use CCA and conclude that price oscillations and exchange rate movements have a strong impact on the option value whereas the other factors have a smaller impact. Finally, Maier (2021) approaches a portfolio of options available and studies the following options: to invest, to mothball operations, to reenter and to permanently shut down activities. The author begins by visiting the case of a copper mine project like the one addressed by Brennan & Schwartz (1985) and derives a solution using the IDSR – Influence Diagram Simulation Regression technique. This is a solution that moves away from considering that a simulation-based approach is more efficient in solving American-style types of investment problems than Monte Carlo-based techniques.

4.2. R&D and Innovation Projects, and Technology Adoption

R&D and Innovation projects are known to imply considerable amounts of irreversible investment, face extremely high levels of uncertainty, and present a cascading option structure. This means that the typical investment setting of an R&D project is a sequential one. Firms invest by stages and assess results and weigh up future decisions based on new relevant information. This setting, therefore, is particularly attractive for ROA and, more specifically, to highlight the importance of project abandonment at the end of each stage. The investment expenses undertaken in the previous stage(s) are sunk costs and the optimal decision derives from comparing the incremental investment needed to complete the next stage and the present value of the expected benefits to be obtained. In fact, Huchzermeir & Loch (2001) argue that it is the presence of the abandonment option that offers the possibility of making the investment in stages, deciding, at each stage, based on the arrival of new information, whether to proceed further or whether to stop[11].

Early contributions to the R&D literature that embrace a sequential investment problem are the ones by Baldwin (1982), Prastacos (1983), Grossman & Shapiro (1986) and Bar-Illan & Strange (1998). Baldwin (1982) addressed the optimal sequential investment by an oligopolistic firm (a price leader). The author examined the impact of demand, industry structure, and technology on

[11] This sequential investment setup is also applicable to venture capital investment problems, which are out of the scope of this work.

the value of the aggregate growth opportunities faced by the oligopolistic firm, which exhibit an expected positive NPV. Prastacos (1983) derives a model that addresses the problem of a firm with access to a limited amount of capital and which makes sequential decisions on long-term investments, under uncertainty regarding the timing or the quality of future opportunities. Grossman & Shapiro (1986) address the same problem, a single firm pursuing an R&D program over time, also in a deterministic setup, but leave the dynamics of oligopolistic interaction out of consideration. Bar-Illan & Strange (1998) analyze a two-stage investment model with time-to-build. The authors present algebraic solutions for an individual firm's optimal sequential investment with costless suspension, without suspension, in the intermediate case of costly suspension (*i.e.,* mothballing) and concerning the aggregate investment. To the best of our knowledge, this is the first paper to address temporary or permanent abandonment under the sequential investment setting.

The most recent relevant contributions for the sequential investment problem are the ones by Pennings & Lint (2000), Schwartz (2004), Koussis et al. (2013), Hauschild & Reimsbach (2015), Hagspiel et al. (2016) and Jou (2018). Pennings & Lint (2001) propose a model to value a phased rollout as, in some circumstances, it seems profitable to postpone a global market introduction to 'learn' about the market, *i.e.,* to receive new valuable information. The state variables are the unit sales margin and the sales path, both assumed to follow a gBm and the investment costs are fixed. Since the option's life is finite, the well-known Black & Scholes (1973) model is applied and an analytical solution is reached, thus enabling firms to know when to rollout the investment and when to abandon the project.

Schwartz (2004) suggests a simulation approach to valuing patents and patented-protected R&D projects under the ROA. The proposed model allows for the possibility of abandoning the project when costs are higher than expected or when expected cash flows turn out to be smaller than anticipated. The author models the investment costs as a gBm (the same diffusion process proposed by Pindyck, 1993), accounting for 'time to build' and technical uncertainty. The other state variable, the expected cash flows, also follows the same diffusion process. To find a solution, the author suggests resorting to numerical methods since the dimensionality problem mentioned in the previous section is present[12]. Finally, using a numerical example, the author concludes that the abandonment option represents a very substantial part of the project value, especially when uncertainty is high, as predicted by option pricing theory. Koussis et al. (2013) resort to probabilistic methods too and propose a multi-period solution for investment in product development, which also allows for valuing preemption and innovation options. The authors construct a numerical lattice, but their work does not consider the abandonment option explicitly. Hauschild & Reimsbach (2015) propose a binomial approach to modelling sequential R&D investments,

[12] The differential system does not allow for homogeneity of degree one and, as such, reducing the solution to just one state variable would be theoretically unsound.

within a compound options' framework. Here, the possibility of abandoning the project is not explicitly considered, albeit the model is structured as a sequence of compound call options and, as such, should not be exercised if 'out-of-the-money'. The authors argue that the binomial solution to modelling the sequential compound option enhances the practical validity of applying the ROA by reducing the mathematical complexity compared to continuous-time analytical option pricing models. Nevertheless, the authors do not address the abandonment option for its own merit and relevance, rather solely by recognizing its 'inverse' option nature to the series of compound call options addressed.

The paper by Hagspiel et al. (2016) analyzes the problem of a firm facing a declining profit stream for its established product, and then assumes that managers have the option to either invest in a new technology and/or to exit. This means that the firm may decide to invest but holds the option to exit at any point in time or may decide to exit the market without undertaking the investment. Both the output price and quantities are governed by gBm, and an inverse demand function is defined to capture the relationship between the two state variables. The authors extend previous works about investing or exiting the market in the presence of declining profit by considering the size of the investment to change in each stage. An optimal solution is derived resorting to DP and the value function for the option value to exit is presented.

Jou (2018) studies the optimal policy for a firm that incurs in a sunk cost with the purpose of innovating, and its success is subject to an exogeneous arrival following a Poisson jump process. Once the firm successfully develops an innovation, it will be granted a patent and will simultaneously commercialize the patent immediately, therefore receiving a reward at each instant that evolves as a gBm. The firm's R&D investment decision to not renew the patent is derived in continuous-time and the solution is obtained by applying numerical methods (more specifically, the finite difference method developed by Hull & White, 1990) since there is no analytically tractable solution for a decision rule expressing a series of complex compound options – the firm only renews the patent if the benefits from current operations exceed the renewal fee, in each period.

4.3. Infrastructure Projects

We find the literature about the value of abandonment option in other business sectors to be scarce. Some exceptions concern the research related to infrastructure projects under Public-Private Partnerships (PPP) structures, such as the ones by Blank et al. (2016), Shan et al. (2010), Brandão & Saraiva (2008), Chiara et al. (2007), Huang & Chou (2006) and Ho & Liu (2002).

Blank et al. (2016) study the possibility of including guarantees or other mechanisms to mitigate the risks for private investors under a PPP agreement. The authors argue that is necessary to value these mechanisms under the ROA to analyze the project's economic feasibility and risk allocation. However, in this setting, the private firm has an implicit option to abandon the project that should be simultaneously assessed. These mechanisms should be calibrated to protect the return rate from the private firm and, therefore, reduce the probability of exercising the option to abandon. The suggested model is based on a binomial tree and the authors obtain the model's outcomes by applying the Monte Carlo simulation. Shan et al.'s (2010) work analyzes the revenue risk in the context of a PPP transportation project and proposes a 'collar option' solution, *i.e.,* a call option and a put option combined. Other studies (*e.g.,* Brandão & Saraiva, 2008; Chiara et al., 2007; Huang & Chou, 2006; Cheah & Liu, 2006; Ho & Liu, 2002) have indicated that a revenue guarantee put option could function as a revenue risk mitigation strategy in PPP projects. Shan et al.'s (2010) model refines the put option approach and suggests a more efficient solution based on the possible exercise of a call option and a put option, when optimal conditions are met.

We do believe that there are opportunities for future research addressing the abandonment solution under a real options framework in business sectors such as manufacturing, tourism, and the broader services industry.

5. The Strategic Role of the Abandonment Option

5.1. Corporate Divestitures

Business unit divestments can occur in a variety of ways—including spin-offs, equity carveouts, and sell-offs (Damaraju et al., 2005). Spin-offs are partial divestments where the equity in the newly formed firm is distributed pro-rata to the existing firm's shareholders. In an equity carve-out, a part of the equity in the newly formed entity is issued to new shareholders. In a sell-off, the business unit is completely sold to another company. Of these modes, spin-offs and equity carve-outs are staged divestments and are often the first stage of a further spinoff/carve-out, complete divestment or sell-off, or the bringing back the unit into the parent company (Zingales, 1995). This means that actions regarding corporate divestments may be seen as the exercise of the option to downscale or the option to abandon, under a real options reasoning.

Strategic management literature has been focusing on divestitures as a type of corporate restructuring, thus emphasizing its strategic importance. Research on resource redeployment proposes that managers should redeploy resources within their firm by withdrawing them from the business where the resources are underemployed and switching them to a business where those resources can be used more efficiently (see, for example, Giarratana & Santaló, 2020; Lieberman et al.,2017; Miller & Yang, 2016; Belderbos et al, 2016; Sakhartov & Folta, 2014). On the other hand, the literature on divestitures advocates that managers should divest businesses containing those resources, thereby completely removing them from the corporate portfolio (Feldman & Sakhartov, 2021). Dranikoff et al (2002) discovered a strong bias against pressure; analyzing more than 200 firms, they found out that more than three-quarters of divestitures are reactive, they are carried-out in response to pressure on the parent firm or on the business unit. Moreover, nearly two-thirds of the reactive divestitures are delayed, occurring only after the parent firm or the unit has suffered from weak performance for several years.

Therefore, these two streams of research address the same underlying strategic problem of how to respond when resources are underutilized within a firm: should the firm redeploy them to other business units in the portfolio or just completely remove them? The latter solution may be viewed as an abandonment problem while the former may be treated as a switching option, under a framework of managing a portfolio of strategic real options.

Damaraju et al. (2015) state that a high level of ownership of a business unit can be conceptualized as the firm taking a put option position on the business unit. Therefore, non-divestment can be thought of as holding the put option, and business unit divestment can be considered as exercising the same put option. These authors stress that both one-time divestment and staged divestment options have value, and both are sensitive to uncertainty, *i.e.*, the higher the uncertainty levels, the greater the value of both options. The work from Kumar & Shyam (2005) also addresses the problem of divesting. Firms may consider it optimal to abandon a joint venture alliance and this study examines the value behind the termination of the venture and the characteristics of the target market. In line with the real options reasoning, the authors conclude that ventures divested to refocus a parent firm's product market portfolio were associated with significant value creation. Moschieri and Mair (2008) argue that a real option analytical perspective to understand companies' investments is also applicable to decisions regarding partial or full divestitures, because the real options logic may serve to manage the uncertainty about the value of the unit to be divested. These authors state that corporate divestitures should be viewed as the exercise of the option to abandon, partially or fully, a business unit. The strategic perspective of such an exercise is one of the most interesting approaches to the strategic corporate restructuring and one of the most innovative ways of dealing with strategic divestment decisions. As observed by Smit &Trigeorgis (2006), to strategic planning is a process of actively developing and managing a portfolio of real options in the context of competitive interactions. We proceed to address the importance of the abandonment option in a dynamic setting.

5.2. The Abandonment Option in a Dynamic Setting

In the last three decades, real options reasoning has become relevant in other fields of knowledge, such as general management theory, operational research, industrial organization, and game theory. Real options have also contributed to framing a new vision of strategic decisions. Luehrman (1998) states that, in financial terms, a business strategy is much more like a series of options than a series of static cash flows. Amaran & Kulatilaka (1999) argue that taking an option-based approach is not simply a matter of using a new set of valuation equations and models – it requires a new way of framing strategic decisions. In a more recent work, Amram & Kulatilaka (2006) highlighted that ROA is a way of thinking that helps managers devise their strategic options (the future opportunities that are created by today's investments), while considering their likely effect on shareholder value.

The real options theory has been increasingly accepted in the strategic management literature as a valid approach to strategic decisions, by embracing in its core the strategic role of managerial flexibility (Trigeorgis & Tsekrekos, 2018; Belderbos et al, 2016; Damaraju et al. 2015; Kumar and Shyam, 2005). Notwithstanding, besides accounting for the presence of real options in firms' strategic management, it is also essential to consider the interaction between the firm's objectives, its competences, and the changes occurring in the marketplace. Given the existence of many situations where economic decisions are made under conditions of conflict, *i.e.*, in which the action of one firm provokes a response from others, and considering that in volatile markets, a firm not only confronts known competitors, but also other uncertainties (such as entry from new competitors, product substitutes or new technologies, all of which can transform the competitive landscape), strategic decisions should account for the interdependence amongst firms and other stakeholders in the industry. Therefore, the literature on real options game emerged as a natural step to properly model the strategic role of real options.

Game theory has made important contributions to strategic management in two areas. First, game theory provides a framework for strategic decisions by defining it as game in terms of players, strategies and the payoffs from every situation that emerges from a combination of all the players' choices in the game, which are the main drivers of the choices, and that take into consideration not only the outcomes but also the players' preferences. The second is the insight into bargaining and competition which may be used to forecast the equilibrium results of competitive conditions, and the impact of the strategic repositioning by another player (Rogers, 2002).

We find surveys dealing with the intersection between real options and game theory (Boyer et al., 2004; Azevedo and Paxson, 2014), investment under uncertainty (Chevalier-Roignant et al., 2011), capacity choice (Huberts et al., 2015), and the applications of real options in operational research (Trigeorgis and Tsekrekos, 2018), with the purpose of identifying possible avenues for future research. As Azevedo and Paxson (2014) stressed, there is a gap in the literature regarding the option to abandon. Usually, papers that apply game theory and the real options theory consider the option to invest, rather than to disinvest. However, some pieces of research draw attention to the exit-reenter decision in a dynamic setting, in line with the above-mentioned stream of literature on strategic resource redeployment. Multinational firms may create a hedge against their exposure to exogenous shocks (such as exchange rate movements), by exercising the option to abandon some business units and reallocate resources to other units (see Botteron et al., 2003; Miller and Reuer, 1998b; Miller and Reuer, 1998a).

The foundations for the models with option to exit were presented by Ghemawat and Nalebuff (1985), Fudenberg and Tirole (1986), and Ghemawat and Nalebuff (1990) in a deterministic fashion. In their setting, firms within a duopoly decide either to remain operating in the market or to exit (Ghemawat and Nalebuff, 1985) or to continuously adjust capacity (Ghemawat and Nalebuff, 1990). Due to the importance of economies of scale, large firms reduce capacity before small firms, and they keep adjusting their size until they reach their rival's. This happens because the marginal revenue of big firms is lower than the revenue of smaller firms, so there are greater benefits to reduce capacity (Ghemawat and Nalebuff, 1990), although even in a 'war of attrition', it is possible for both firms to remain in the market if their fixed and opportunity costs are low enough (Fudenberg and Tirole, 1986).

While Smets (1993) still constitutes an important reference for the combination of real options and game theory in a continuous-time model with strategic product-market competition, and Dixit (1989) for modeling entry and exit decisions, the first stochastic duopoly model with disinvestment analyzing a declining market was Fine and Li (1989) in a discrete-time framework[13]. They find that demand discrete changes allow room for non-unique exit sequences (*i.e.,* multiple equilibria). This is due to the possibility of a jump from a point where the duopoly is viable to points where only a monopoly is viable or neither firm is viable. If there is enough asymmetry in the payoffs between the firms and/or the demand jumps are small, there may exist a unique equilibrium.

In a continuous-time set-up with declining demand, Sparla (2004) analyzes a second-mover advantage given the impact of the disinvestment of the leader on profits. On the one hand, large enough variable cost differences will ensure that one firm waits until the other firm leaves and therefore the model has only one equilibrium. On the other hand, in line with Fine and Li (1989), if the heterogeneity in costs is low, multiple equilibria arise. Moreover, the strategic externalities become less relevant when price volatility increases.

Finally, in the next section, we provide some concluding remarks about the role of the abandonment option in strategic capital allocation, along with some avenues for future research.

6. Concluding Remarks and Future Research

When considered on a stand-alone basis, the option to abandon has been receiving less attention from researchers than the option to invest.

Moreover, the investment decision should be affected by the possibility of early abandonment, *i.e.,* prior to the end of the assets' useful life, and at the project's appraisal stage. The fact that most research pieces address the abandonment option included in a broader capital investment setting is difficult to explain, since other options – most notably, the option to delay the project's implementation – are frequently treated individually. One of the reasons may reside on the importance of addressing the problem from a compound options' perspective.

[13] Please see Smith and Ankum (1993) for a discrete-time model with real options and principles of game theory and industrial organization.

However, this argument also applies to other options, such as the optimal timing to invest and, in these cases, researchers seem to handle well the fact that this option has its own merit and relevance. Nevertheless, as we have been mentioning throughout this work, several research pieces address the abandonment solution, alone or combined with other types of options. The natural resources industry, R&D and innovation and technology adoption are the most active in such context. We do believe that there is room for more research in other business sectors, such as manufacturing, infrastructure projects (where the ROA is applied but where the abandonment option is clearly misrepresented), real estate, tourism industry, the services sector, amongst others.

Far from being an exhaustive survey of the literature, this paper, therefore, leaves room for reviewing the abandonment option beyond the conventional investment project-style setting. The decision-making process of exiting from a certain position after some effort was undertaken and where uncertain conditions are present regarding the probable outcome is an ongoing process that presents attractive opportunities for future research. The ROA is applied in research aiming at modeling agency problems, contract designing and human capital topics, amongst other decision problems. Its interaction with operational research, industrial organization and game theory opens a broader avenue for interconnected research under the ROA.

In addition, we believe that a broader set of applications of the tools provided by game theory can be applied to research questions related to the abandonment option. Usually, papers applying both game theory and the real option to abandon a project study the interaction between two firms, not only in competitive duopolies such as the characteristics of competitive thresholds to exit present in Goto et al. (2008) and, depending on the degree of asymmetry or the debt-financing structure, in Murto et al. (2004) and Lambrecht (2001) respectively, but also concerning Stackelberg-like type of games in supply chains as demonstrated by Burnetas & Ritchken (2005) or R&D alliances (Morreale et al, 2017). Therefore, the literature could develop additional knowledge in this area by developing models with more complex market structures and/or a greater number of players.

The aforesaid duopoly models focus on stationary (partial) equilibrium, hence they do not consider the path that leads to the equilibrium. Also, the general equilibrium effects from and as a result of the real options of the firms are not addressed. Furthermore, after a shock, how do firms respond within a given sector? These are unanswered questions that could be addressed in the future. Finally, the embryonic literature on the real options game applied to the option to abandon is mainly theoretical. Therefore, econometric modeling and experimental and/or quasi-experimental evidence are needed. One exception is the lab experiment with undergraduate students on the impacts of acquiring information under uncertainty on the option to abandon investments, conducted by Bragger et al (2008).

It is clear at this point that when considering the strategic role of capital allocation, management decisions should embody the real options theory to assess and consider the value from the flexibility (or to identify the areas in which not being flexible implies that the firm is losing potential) arising from its basket of interrelated real options. Firms succeed by finding the right place and, especially, the right time to invest, but they are more likely to thrive if they take the option to abandon and the real options approach more carefully into consideration.

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Appendix: Selected Literature of the Abandonment Option*

* Listed by alphabetical order.

Articles	Output	State Variable(s)	Diffusion Process(es)	Formalism	Methodology Applied	Additional Options	Industry / Investment Project
Abadie & Chamorro (2017)	Oil.	Price. Cost per unit. Reserve quantity.	Price follows a IgBmi. Cost per unit follows a gBm. Reserve quantity is exponentially decayed.	(i) Continuous time; (ii) Discrete time.	(i)Analytical methods in continuous time; (ii) Monte Carlo Simulation combined with least squares regression.	Option to Delay. Options to mothball, and to reenter.	Natural Resources /Oil extraction.
Baldwin (1982)	N/A	Industry capacity.	Markov process.	Continuous time.	Dynamic Programming.	Sequential investment.	General applicability.
Bar-Illan & Strange (1998)	N/A	Price.	gBm.	Continuous time.	Dynamic Programming. Analytical Methods.	Sequential investment. Option to suspend.	General applicability.
Blank et al (2016)	N/A	Traffic Process.	gBm.	Continuous time.	Analytical methods.	None.	Infrastructure / PPP
Brennan & Schwartz (1985)	Copper.	Price. Mine Inventory.	Price follows a gBm. Inventory is a binary variable.	Continuous time.	Numerical methods.	Exit and reenter options.	Natural resources / copper mine.
Clarke & Reed (1990)	Oil.	Price. Production Rate.ii	Price follows a gBm. Production Rate follows a gBm.	Continuous time.	Dynamic programming. Analytical methods.	None.	Natural resources / general applicability.

Articles	Output	State Variable(s)	Diffusion Process(es)	Formalism	Methodology Applied	Additional Options	Industry / Investment Project
Cortazar et al (2001)	Oil or any other commodity.	Price. Risk Factor.	Price follows a gBm. Risk Factor follows a simple Brownian motion.	Continuous time.	Numerical Methods.	Option to (sequentially) invest.	Natural resources / general applicability.
Cox & Martin (1983)	N/A	1.Project value. Salvage value. Discount rate.	gBm.	Continuous time.	Analytical methods.	None.	General applicability.
De et al (1983)	N/A	1.Project's Cash Flows 2. Salvage value.	Both variables follow a Markov process.	Discrete time.	Dynamic programming. Probabilistic methods.	None.	General applicability.
Dixit (1989)	N/A	Price. State of the firm.	Price follows a gBm. State of the firm is a binary variable.	Continuous time.	Numerical methods.	Option to suspend and to reenter.	General applicability.
Dixit & Pindyck (1994)	N/A	Price. Salvage value	gBm.	Continuous time.	Numerical methods	Option to suspend and to reenter. Option to mothball.	General applicability.
Gibson & Schwartz (1990)	Oil.	Price. Convenience yield.	gBm.	Continuous time.	Analytical methods.	None.	Natural resources / General applicability.

Articles	Output	State Variable(s)	Diffusion Process(es)	Formalism	Methodology Applied	Additional Options	Industry / Investment Project
Guedes & Santos (2016)	Oil and Gas.	Price. Reserve size.	Price modelled first as gBm and after a mean reverting process. Reserve size is a discrete variable.	Discrete time.	Probabilistic methods; binomial, trinomial, quadrinomial and hexanomial lattices.	Option to expand. Option to downscale.	Natural resources / Oil and gas field project.
Hauschild & Reimsbach (2015)	N/A	Project's expected cash flows.	Two-state discrete process.	Discrete time.	Binomial model.	Sequential investment. Compound Options.	R&D and innovation.
Hagspiel et al (2016)	N/A	Market demand.	gBm.	Continuous time.	Dynamic programming. Numerical methods.	Option to invest.	Technology adoption
Huang & Chou (2006)	N/A	Project revenues.	gBm.	Continuous time.	Analytical methods.	None.	Infrastructure / PPP.
Kensinger (1980)	N/A	Project value.	gBm.	Continuous time.	Analytical methods.	None.	General applicability.
Lumley & Zervos (2001)	Oil or any other commodity.	Price. Project status (active or closed)	Price follows a gBm. Project status is a binary variable.	Continuous time.	Dynamic programing. Analytical methods.	(Sequential) entry and exit.	Natural resources / General applicability
Myers & Madj (1980)	N/A	Cash Flows. Salvage value	gBm.	Continuous time.	Analytical methods.	None.	General applicability.
McDonald & Siegel (1985)	N/A	Price. Salvage value.	gBm.	Continuous time.	Analytical methods.	None.	General applicability.
Olsen & Stensland (1988)	Any extracted commodity.	Price. Quantities.	Price follows a gBm. Quantities follow a gBm.	Continuous time.	Analytical methods.	None.	Natural resources / General applicability.

Articles	Output	State Variable(s)	Diffusion Process(es)	Formalism	Methodology Applied	Additional Options	Industry / Investment Project
Pennings & Lint (2000)	N/A	Unit sales margin. Sales path	gBm.	Continuous time.	Analytical methods.	Option to invest. Option of phased rollout.	Technology adoption / General applicability.
Prastacos (1983)	N/A	Expected return.	gBm.	Continuous time.	Dynamic programming.	Sequential investment	General applicability.
Schwartz (2004)	N/A	Cash Flows. Investment cost.	Cash Flows follow a gBm. Investment cost follows a gBm with jumps.	Discrete time.	Probabilistic methods. Monte Carlo simulation.	Sequential investment.	Pharmaceutical Industry / R&D Intensive Projects.
Zhou et Chao. (2020)	Oil.	Project value. Price. Exchange Rate. Political environment.	All variables follow Markov processes.	Discrete time.	Probabilistic methods; Trinomial lattice.	None.	Natural resources / Oilfield project

IgBm stands for Integrated geometric Brownian motion, a stochastic process that also allows the price to mean revert.
The two variables are substituted by just one – the revenue, which is equal to the price multiplied by the quantity extracted.
iii The authors apply the IDSR (influence diagram simulation and regression) approach.